

Time Value of Money

A dollar you receive today is worth more than a dollar you may receive a year from today! Money has a time value because interest rates are positive. For example, if you earn 5 percent per year on your savings account, one dollar will grow to one dollar plus five cents after one year. Since the present value of \$1.05 to be received one year from now (if interest rates are 5%) is \$1.00, then the present value of \$1.00 to be received one year from now (again if interest rates are 5%) must be some value less than \$1.00. In fact, the present value can be calculated using the formula

$$PV = FV / (1 + r)^n$$

where

PV is present value

FV is future value

r is the rate of interest per period

n is the number of compounding periods (per year).

Using the formula for our example:

$$PV = \$1.00 / (1.05)^1$$

$$PV = \$0.95.$$

Today's value of \$1.00 to be received one year from now if the interest rate is 5 percent is \$0.95.

Business executives must consider the time value of money when making business investment spending decisions. They know that future profit projections must be converted to the present value in order to make a correct decision about whether a certain business project is profitable. Notice that the interest rate is in the denominator of the formula indicating the present value is inversely related to the interest rate. Thus, less business investment spending is worthwhile at higher interest rates.

For example, assume a business was considering the purchase of a new machine that costs \$2,000 now. The machine is expected to generate profits of \$1,000 at the end of year one and \$1,400 at the end of year two. For simplicity, assume the machine completely wears out and is worthless after the two years. Also assume the business must borrow the \$2,000 at 9 percent interest. Should the business borrow and purchase the machine?

Using the present value formula:

$$PV = \$1,000 / (1.09)^1 + \$1,400 / (1.09)^2 = \$917.43 + \$1,178.35 = \$2,095.78.$$

The business should invest in the machine since the present value of its future profits from the machine is greater than the cost of the machine: $\$2,095.78 - \$2,000 = \$95.78$.

Now, what if the rate the business had to pay to borrow increased to 15 percent?

Using the present value formula:

$$PV = \$1,000 / (1.15)^1 + \$1,400 / (1.15)^2 = \$869.57 + \$1,058.60 = \$1,928.17.$$

The business should not invest in the machine since the present value of its future profits from the machine is now less than the cost of the machine: $\$1,928.17 - \$2,000 = (\$71.83)$.

Understanding the time value of money also helps for understanding the relationship between bond prices and interest rates. A bond is a loan with a fixed interest rate called the coupon rate. Bonds are long-term fixed-rate loans of usually 20 or 30 years. The seller (borrower) of a bond agrees to pay the buyer (lender) the amount of interest specified each year plus the face value of the bond at the end of the specified period, again typically 20 or 30 years. Often the buyer of the bond (lender) incurs a liquidity problem and needs to sell the bond before it reaches its maturity. So, at what price can the owner of the bond sell the bond?

To answer that question, let's assume the original bond was a 20-year bond with a face value of \$1,000 and the coupon rate was 5 percent. That means the owner was receiving \$50 in interest payments each year and was planning on receiving the \$1,000 back at the end of year 20. But, let's further assume the owner needs some cash and wants to sell the bond after owning it for 18 years and that current interest rates for bonds with the same level of risk are now 7 percent. That means there are two more interest payments due (one next year and one two years from now) and the face value will be due at the date of maturity or 20th year (two years from now). What price can the owner sell the bond for now that current interest rates are higher?

Using the present value formula:

$$PV = \$50 / (1.07)^1 + \$1,050 / (1.07)^2 = \$46.73 + \$917.11 = \$963.84.$$

Note: Current interest rate is higher and the price of the bond is lower.

Now assume that current interest rates for bonds with the same level of risk are now 3 percent, which is lower than the 5 percent coupon rate. So, now what price can the owner sell the bond?

Using the present value formula:

$$PV = \$50 / (1.03)^1 + \$1,050 / (1.03)^2 = \$48.54 + \$1,019.42 = \$1,067.96.$$

Note: Current interest rate is lower and the price of the bond is higher. We can conclude that bond prices are inversely related to interest rates.

Final note: What would the same \$1,000 bond sell for if interest rates were still equal to the 5 percent coupon rate? Hopefully, you concluded that the price would be the same as the original price or \$1,000. To check this out using our formula:

$$PV = \$50 / (1.05)^1 + \$1,050 / (1.05)^2 = \$47.62 + \$952.38 = \$1,000.$$

1. What will \$3,000 deposited into a savings account be worth after one year if interest rates are 3 percent compounded yearly?

2. What will \$3,000 deposited into a savings account be worth after two years if interest rates are 3 percent compounded yearly?
3. What is the present value of \$3,000 you are scheduled to receive one year from today if you are currently earning 3 percent on your savings account?
4. What is the present value of \$3,000 you are scheduled to receive two years from today if you are currently earning 3 percent on your savings account?
5. Assume you have owned a 20-year \$1,000 bond with a coupon rate of 6 percent for 17 years. If current interest rates on similar bonds are 9 percent, at what price could you sell the bond today?
6. Assume a business is deciding whether to invest in a new project that is projected to generate profits of \$90,000 each year for the next three years. The project start-up costs are \$225,000.
 - (A) If the business normally earns 11 percent on its investments, should the business invest? Show/explain.
 - (B) If the business normally earns 5 percent on its investments, should the business invest? Show/explain.